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### Optimization of Evaporator to Condenser

# Optimization of Evaporator to Condenser Length Ratio for Heat Pipe Maximum Performance

**Thermal Conductivity:** A measure of the ability of a material to transfer heat. Given two surfaces on either side of a material with a temperature difference between them, the thermal conductivity is the heat energy transferred per unit time and per unit surface area, divided by the temperature difference [1].

## Introduction

Heat pipes have a multitude of applications in the thermal management of electronics. The thermal resistance of a heat pipe strongly depends, among other parameters, on so-called L-ratio, which is the ratio between the evaporator length and condenser length. The optimal L-ratio is defined as being the value for which the heat pipe thermal resistance is minimal, for a certain heat pipe assembly configuration. Finding the optimal L-ratio offers a valuable tool for thermal practitioners to most effectively deploy the heat pipe in their application.

This article reviews an analytical approach for finding this optimum L-ratio [1]. The formula obtained is applied to round heat pipes frequently used in electronics cooling.

## Nomenclature

$a_f$  = annular fin surface area,  $\text{m}^2$

$A$  = constant

$A_f$  = is heat transfer area at condenser section, finned heat pipe,  $\text{m}^2$

$A_p$  = is heat transfer area at condenser section, bare heat pipe,  $\text{m}^2$

$C_1, C_2, C_3, C_4, C_5, C_6, C_7$  = constants

$CA, CB$  = compounded constants

$d_i$  = heat pipe inner diameter,  $\text{m}$

$d_o$  = heat pipe outer diameter,  $\text{m}$

$d_{iw}$  = wick structure inner diameter,  $\text{m}$

$d_{ow}$  = wick structure outer diameter,  $\text{m}$

$d_{me}$  = mean diameter of bonding material at evaporator section,  $\text{m}$

$d_{mc}$  = mean diameter of bonding material at condenser section,  $\text{m}$

$D$  = annular fin diameter or equivalent diameter of a rectangular fin,  $\text{m}^2$

$h$  = equivalent heat transfer coefficient of bare pipe to finned heat pipe at condenser section,  $\text{W}/\text{m}^2 \cdot ^\circ\text{C}$

$h_f$  = heat transfer coefficient of finned heat pipe at condenser section,  $\text{W}/\text{m}^2 \cdot ^\circ\text{C}$

$k_{eff}$  = effective thermal conductivity of liquid saturated wick structure,  $\text{W}/\text{m} \cdot ^\circ\text{C}$

$k_p$  = thermal conductivity of pipe wall,  $\text{W}/\text{m} \cdot ^\circ\text{C}$

$K_b$  = thermal conductivity of heater block material,  $\text{W}/\text{m} \cdot ^\circ\text{C}$

$K_{bm}$  = thermal conductivity of bonding material,  $\text{W}/\text{m} \cdot ^\circ\text{C}$

$L$  = total heat pipe length,  $\text{m}$

$L_a$  = length of adiabatic section,  $\text{m}$

$L_c$  = length of condenser section,  $\text{m}$

$L_e$  = length of evaporator section,  $\text{m}$

$L_h$  = length of heat source,  $\text{m}$

$P$  = fin pitch,  $\text{m}$

$R$  = total thermal resistance of the heat pipe,  $^\circ\text{C}/\text{W}$

$R_{sp}$  = spreading thermal resistance at the evaporator section,  $^\circ\text{C}/\text{W}$

$R_{eb}$  = heater block thermal resistance,  $^\circ\text{C}/\text{W}$

$R_{ebm}$  = thermal resistance of bonding material at evaporation section,  $^\circ\text{C}/\text{W}$

$R_{ep}$  = thermal resistance of radial conduction of pipe wall at evaporation section,  $^\circ\text{C}/\text{W}$

$R_{ew}$  = thermal resistance of radial conduction of liquid/wick combination at evaporation section,  $^\circ\text{C}/\text{W}$

$R_{ee}$  = vaporization thermal resistance,  $^\circ\text{C}/\text{W}$

$R_{cc}$  = condensation thermal resistance,  $^\circ\text{C}/\text{W}$

$R_{cw}$  = thermal resistance of radial conduction of liquid/wick combination at condenser section,  $^\circ\text{C}/\text{W}$

$R_{cp}$  = thermal resistance of radial conduction of pipe wall at condenser section,  $^\circ\text{C}/\text{W}$

$R_{ebm}$  = thermal resistance of bonding material at condenser section,  $^\circ\text{C}/\text{W}$

$R_{sk}$  = thermal resistance of forced air convection at heat sink to ambient,  $^\circ\text{C}/\text{W}$

$R_v$  = vapor thermal resistance,  $^\circ\text{C}/\text{W}$

$R_{aw}$  = thermal resistance of axial conduction of liquid/wick combination,  $^\circ\text{C}/\text{W}$

$R_{ap}$  = thermal resistance of axial conduction of pipe wall,  $^\circ\text{C}/\text{W}$

S = length or width of an equivalent square heat source, m

$S_f$  = fin to fin gap, m

$t_b$  = thickness of the heater block at the evaporator section, m

$t_f$  = fin thickness, m

$t_{bm}$  = thickness of bonding material, m

$W_b$  = width of the heater block, m

$W_h$  = width of the heat source, m

## Heat Pipe Thermal Resistance

A typical cylindrical heat pipe assembly is shown in Figure 1 [1]. The assembly consists of a heat pipe with heating and cooling sections attached to it.

The length of the evaporator ( $L_e$ ) and the length of the condenser ( $L_c$ ) can be varied.

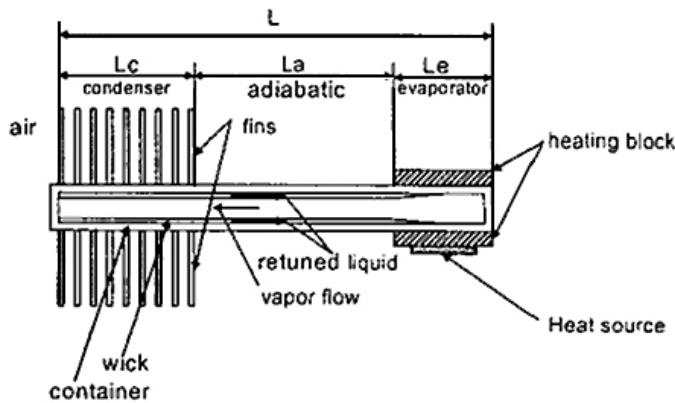


Figure 1. Schematic of a Heat Pipe Assembly [1]

The thermal resistance network for the above assembly is presented in Figure 2.

Table 1 includes order of magnitude estimations of several thermal resistances from the network in Figure 2.

Thermal Resistance	°C/W
$R_{ep}$ and $R_{cp}$	$10^{-1}$
$R_{ew}$ and $R_{cw}$	$10^{+1}$
$R_{ee}$ and $R_{cc}$	$10^{-5}$
$R_v$	$10^{-8}$
$R_{ap}$	$10^{+2}$
$R_{aw}$	$10^{+4}$

Table 1. Magnitude Comparison of Thermal Resistances in Heat Pipes [2]

By analyzing the values in Table 1, the circuits including Rap and Raw can be eliminated, since both these resistances are significantly larger than the others. Also, since Ree, Rcc and Rv are relatively small, they can be neglected. Therefore, the total thermal resistance of the heat pipe (R) can be expressed as:

$$R = R_{sp} + R_{eh} + R_{ebm} + R_{ep} + R_{ew} + R_{cw} + R_{cp} + R_{chm} + R_{sk} \quad (1)$$

The factors on the right hand side can be expressed as:

$$R_{sp} = C_1 \ln(L_e) + A \quad (2)$$

$$R_{eb} = \frac{t_b}{Le \cdot Wb \cdot Kb} \quad (3)$$

$$R_{ebm} = \frac{t_{bm}}{1 + \pi \cdot d \cdot K} \quad (4)$$

$$R_{ep} = \frac{1}{2 \cdot \pi \cdot k_p \cdot L_e} \ln \left( \frac{d_o}{d_i} \right) \quad (5)$$

$$R_{ew} = \frac{1}{2 \cdot \pi \cdot k_{eff} \cdot L_e} \ln \left( \frac{d_{ow}}{d_{iw}} \right) \quad (6)$$

$$R_{cw} = \frac{1}{2 \cdot \pi \cdot k_{eff} \cdot L_c} \ln \left( \frac{d_{ow}}{d_{lw}} \right) \quad (7)$$

$$R_{cp} = \frac{1}{2 \cdot \pi \cdot k \cdot L} \ln \left( \frac{d_o}{d_i} \right) \quad (8)$$

$$R_{cbm} = \frac{1_{bm}}{L \cdot \pi \cdot d_{bm} \cdot K_{bm}} \quad (9)$$

$$R_{sk} = \frac{1}{A_s \cdot h} = \frac{1}{L_s \cdot \pi \cdot d_s \cdot h} \quad (10)$$

Figure 2. Thermal Resistance Network for a Heat Pipe Assembly [1]

For a given heater block and heat source size, the spreading resistance  $R_{sp}$  will be only a function of  $L_e$ , hence equation (2). The equivalent heat transfer coefficient at the condenser section can be

$$h = \frac{A_f \cdot h_f}{A_p} \quad (11)$$

expressed as:

Assuming the fins at the condenser section are annular, the total area for a fin (neglecting fin tip

$$A_f = \frac{\pi}{2} \cdot (D^2 - d_o^2) \quad (12)$$

area) can be calculated as:

The total fin and pipe areas are, respectively:

$$A_f = \frac{L_c}{p} \left[ \frac{\pi}{2} \cdot (D^2 - d_o^2) + \pi \cdot d_o \cdot S_f \right] \quad (13)$$

$$A_p = \pi \cdot d_o \cdot L_c \quad (14)$$

Some geometrical parameters mentioned above are illustrated in Figure 3.

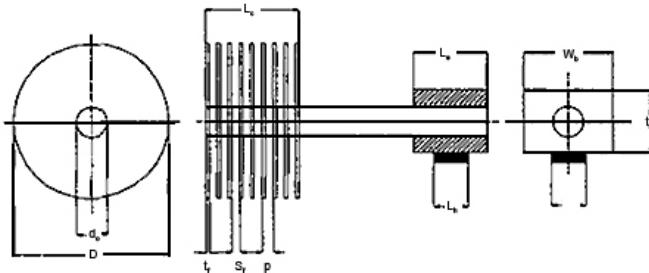


Figure 3. Geometrical Parameters for Heat Pipe Assembly Analysis [1]

Using (13) and (14), equation (11) can be

$$h = \frac{\frac{\pi \cdot L_c}{p} \left[ \frac{1}{2} (D^2 - d_o^2) + D \cdot t_f \cdot d_o \cdot S_f \right] \cdot h_f}{\pi \cdot d_o \cdot L_c} \quad (15)$$

$$= \frac{\left[ 0.5 (D^2 - d_o^2) + D \cdot t_f \cdot d_o \cdot S_f \right] h_f}{d_o \cdot p}$$

rearranged as:

Therefore, the overall thermal resistance can be written as:

$$R = [C_1 \cdot \ln(L_e) + A] + \frac{C_2 + C_3 + C_4 + C_5 + C_6}{L_e} + \frac{C_6 + C_5 + C_4 + C_3 + C_7}{L_c} \quad (16)$$

So, for a specific heat pipe included in a specific assembly we can write:

$$R = F(L_e, L_c) \quad (17)$$

Since

$$L_c = L - (L_a + L_e) \quad (18)$$

equation (18) can be expressed as well as:

$$R = F[L_e, L(L_e)] \quad (19)$$

### Optimal L-ratio

Optimal L-ratio ( $L_e/L_c$ ) is attained when heat pipe thermal resistance is minimal. Therefore, by setting:

$$\frac{dR}{dL_e} = 0 \quad (20)$$

the minimum of the function will be found. Figure 4 illustrates equation (17) between two set values for evaporator length.

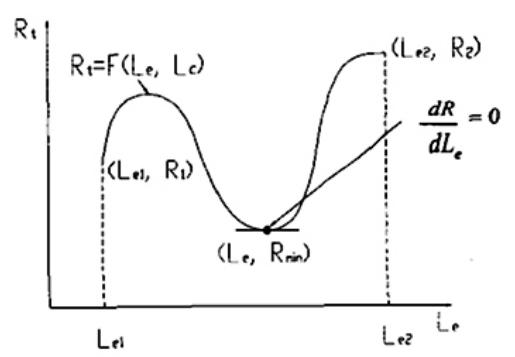


Figure 4. Minimum Thermal Resistance as a Function of Evaporator Length [1]

After appropriate manipulations equation (20) yields:

$$A_f = \frac{C_A}{L_c^2} = \frac{C_b - C_1 \cdot L_e}{L_e^2} \quad (21)$$

or

$$L_e = \left[ L_c \left( C_1 L_c + 4C_A C_B \right)^{0.5} - L_c C_1 \right] / 2C_A \quad (22)$$

where:

$$C_1 = 0.0179 W^{-0.1244} \quad 2.564e^{-132.92tb} \quad 0.0863 S^{-0.535} K_b / 390 \quad (23)$$

$$S = (L_h W_h)^{0.5} \quad (24)$$

$$C_A = C_3 + C_4 + C_5 + C_6 + C_7 \quad (25)$$

$$C_b = C_2 + C_3 + C_4 + C_5 + C_6 \quad (26)$$

The analytical expression obtained above was applied for round heat pipes with outer diameters of 4, 5 and 6 mm and different condenser lengths. Table 2 includes results for a 4 mm OD heat pipe, 10 x 10 mm heat source and a heater block 30 x 30 x 6 mm.

where  $C_2, C_3, C_4, C_5, C_6, C_7$  are constants for a particular heat pipe assembly [1].

At the condenser end, fins were 0.3 mm thick, fin gap 1 mm and fin diameter 40 mm. The optimal  $L_e/L_c$  varies between 0.4 and 0.6.

$L_c$ (m)	$L_e$ (m)	$C_1$	$C_A$	$C_B$	$C_A/L_c$	$C_B - C_1^* L_e)/L_e$	$L_e/L_c$
0.200	0.075	0.032	0.012	0.004	0.30	0.3	0.4
0.100	0.046	0.032	0.012	0.004	1.21	1.21	0.5
0.060	0.030	0.032	0.012	0.004	3.37	3.37	0.5
0.040	0.021	0.032	0.012	0.004	7.58	7.58	0.5

Table 2. Optimal  $L_e/L_c$  for 4 mm OD Heat Pipe Assembly [1]

0.020	0.011	0.032	0.012	0.004	30.34	30.33	0.6
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Table 3 includes results for a 5 mm OD heat pipe, 10 x 10 mm heat source and a heater block 35 x 35 x 7 mm. Fins were 0.3 mm thick, fin gap 1.2 mm and fin diameter 40 mm. Similarly, the optimal varies between 0.4 and 0.5.

$L_c$ (m)	$L_e$ (m)	$C_1$	$C_A$	$C_B$	$C_A/L_c$	$C_B - C_1^* L_e)/L_e^2$	$L_e/L_c$
0.200	0.071	0.028	0.011	0.003	0.27	0.27	0.4
0.100	0.044	0.028	0.011	0.003	1.06	1.06	0.4
0.060	0.029	0.028	0.011	0.003	2.95	2.95	0.5
0.040	0.020	0.028	0.011	0.003	6.64	6.64	0.5
0.020	0.011	0.028	0.011	0.003	26.54	26.54	0.5

Table 3. Optimal  $L_e/L_c$  for 5 mm OD Heat Pipe Assembly [1]

The results for a 6 mm OD heat pipe are presented in Table 4. The heat source was 10 x 10 mm and the heater block 40 x 40 x 8 mm. Fins were 0.4 mm thick, fin gap 1.8 mm and fin diameter 40 mm. In this case the optimal varies between 0.3 and 0.5.

$L_c$ (m)	$L_e$ (m)	$C_1$	$C_A$	$C_B$	$C_A/L_c$	$C_B - C_1^* L_e)/L_e^2$	$L_e/L_c$
0.200	0.068	0.025	0.012	0.003	0.29	0.29	0.3
0.100	0.041	0.025	0.012	0.003	1.15	1.17	0.4
0.060	0.027	0.025	0.012	0.003	3.21	3.21	0.4
0.040	0.019	0.025	0.012	0.003	7.22	7.22	0.5
0.020	0.010	0.025	0.012	0.003	28.87	28.87	0.5

## Conclusions

An analytical formula for calculating the optimal evaporator to condenser ratio ( $L_e/L_c$ ) has been deduced. The formula is a valuable tool for thermal engineers, offering a simple rule for choosing appropriate  $L_e$  and  $L_c$  values. The formula was applied for several frequently used cylindrical heat pipes, yielding optimal L-ratios between 0.3 and 0.6. However, these results are valid for a fixed value of  $L_e + L_c$ . For maximum heat transfer rate (minimum thermal resistance),  $L_e + L_c$  should equal the heat pipe length, which means no adiabatic section. Also, the formula needs to be validated through experimental results and generalized (for different heat pipe cross sections and heat pipe assembly orientations).

## References:

1. Wu, X.P., Mochizuki, M., Saito, Y., Nguyen, T., Wuttijumnong, V., and Wu, D., "Analyzing and Modeling on Optimized L-Ratio of Evaporator Section to Condenser Section for Micro Heat Pipe Heat Sinks", 19th IEEE SEMI-THERM Symposium, 2003.
2. Peterson, G.P., "An Introduction to Heat Pipes, Modeling, Testing and Applications", John Wiley & Sons, Inc., pp.147-150, 1994.

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